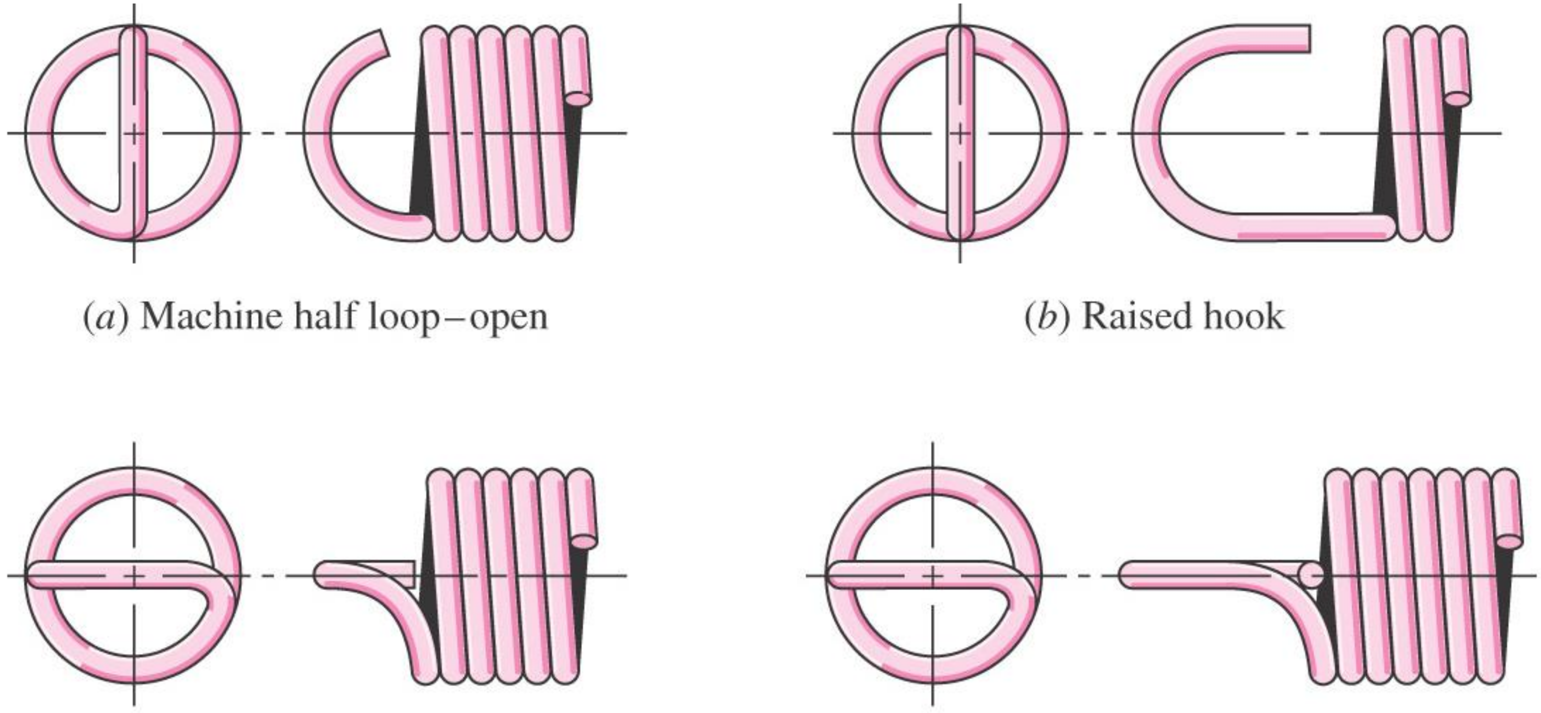
Extension & Torsion Springs (Chapter 10)

Extension Springs

- Extension springs are similar to compression springs within the body of the spring.
- To apply tensile loads, hooks are needed at the ends of the springs.
- Some common hook types:



(*a*) Machine half loop–open

(*b*) Raised hook

(*c*) Short twisted loop

(*d*) Full twisted loop

Fig. 10–5

Normal Stress in the Hook vs. Shear Stress in Body

- In a typical hook, a critical stress location is at point A, where there is bending and axial loading.

$$\sigma_A = F\left[(K)_A \frac{16D}{\pi d^3} + \frac{4}{\pi d^2}\right] \qquad (10\text{–}34)$$

- $(K)_A$ is a bending stress-correction factor for curvature

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} \qquad C_1 = \frac{2r_1}{d} \qquad (10\text{–}35)$$

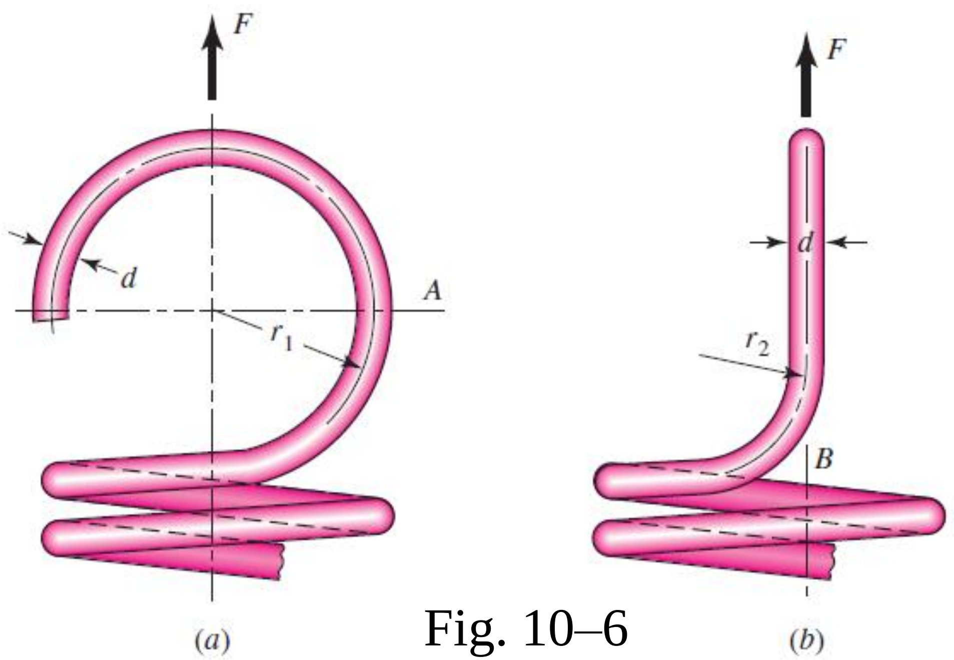


Fig. 10–6

Stress in the Hook

- Another potentially critical stress location is at point *B*, where there is primarily torsion.

$$\tau_B = (K)_B \frac{8FD}{\pi d^3} \qquad (10\text{–}36)$$

- $(K)_B$ is a stress-correction factor for curvature.

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} \qquad C_2 = \frac{2r_2}{d} \qquad (10\text{–}37)$$

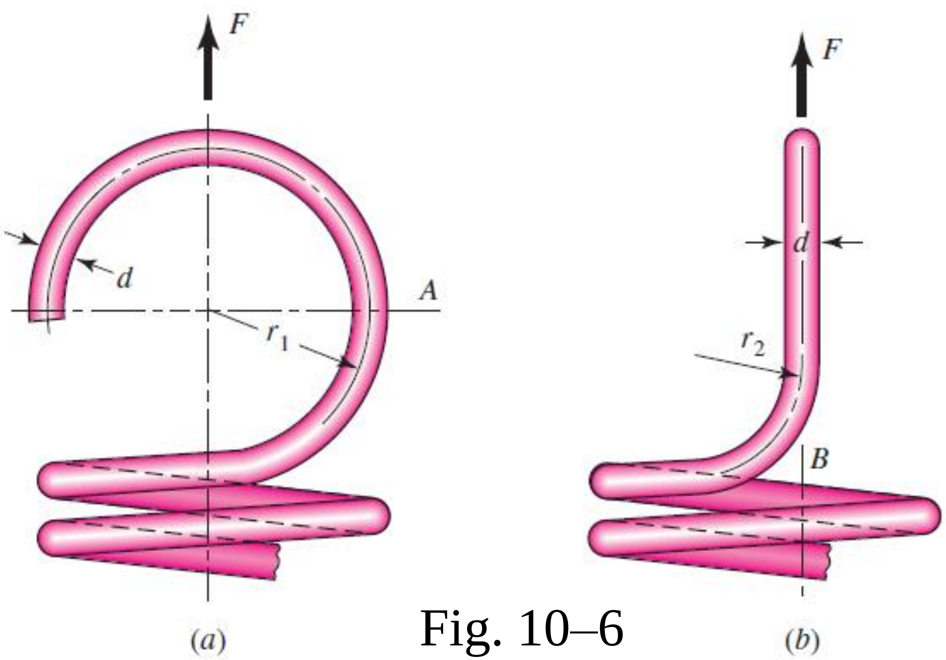


Fig. 10–6

Close-wound Extension Springs

- Extension springs are often made with coils in contact with one another, called *close-wound*.
- Including some initial tension in close-wound springs helps hold the free length more accurately.
- The load-deflection curve is offset by this initial tension F_i

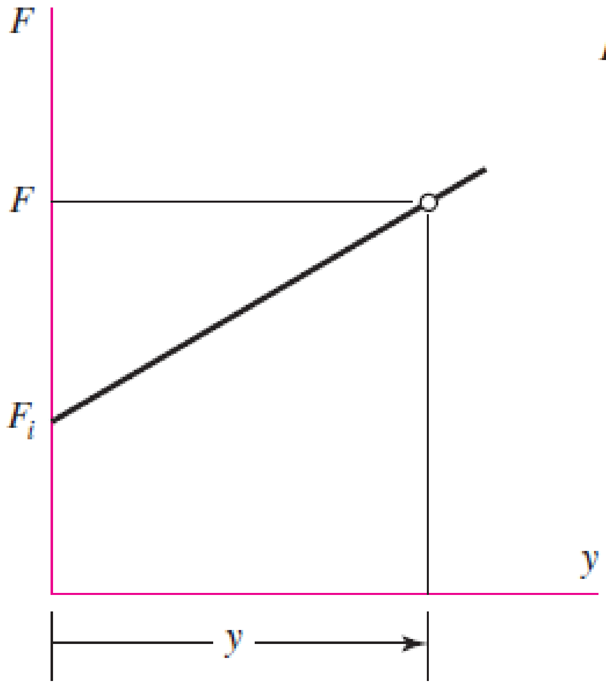


Fig. 10–7 (*a*)

$$F = F_i + ky \tag{10–38}$$

Terminology of Extension Spring Dimensions

- The free length is measured inside the end hooks.

$$L_0 = 2(D - d) + (N_b + 1)d = (2C - 1 + N_b)d \qquad (10\text{–}39)$$

- The hooks contribute to the spring rate. This can be handled by obtaining an equivalent number of active coils.

$$N_a = N_b + \frac{G}{E} \qquad (10\text{–}40)$$

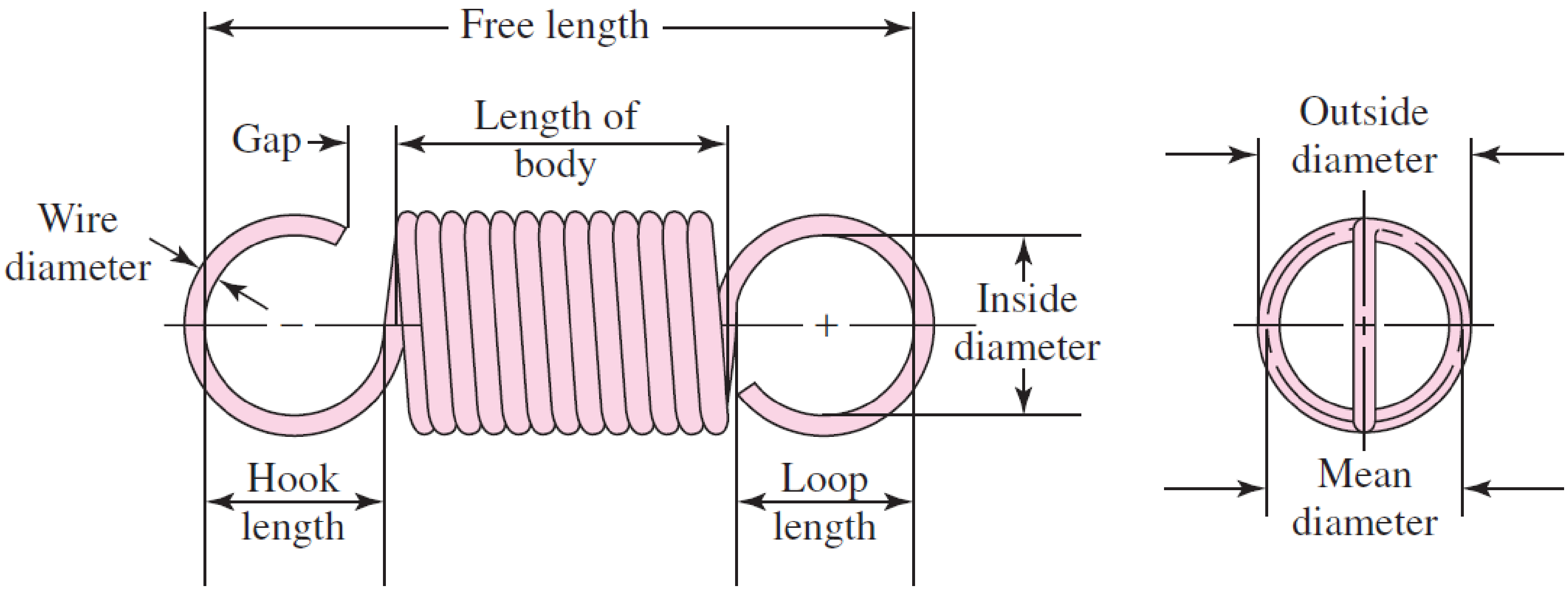


Fig. 10–7 (*b*)

Helical Spring: Coiled Extension Spring

- Similar to compressions springs, but opposite direction
- Equilibrium forces at cut section anywhere in the body of the spring indicates direct shear and torsion

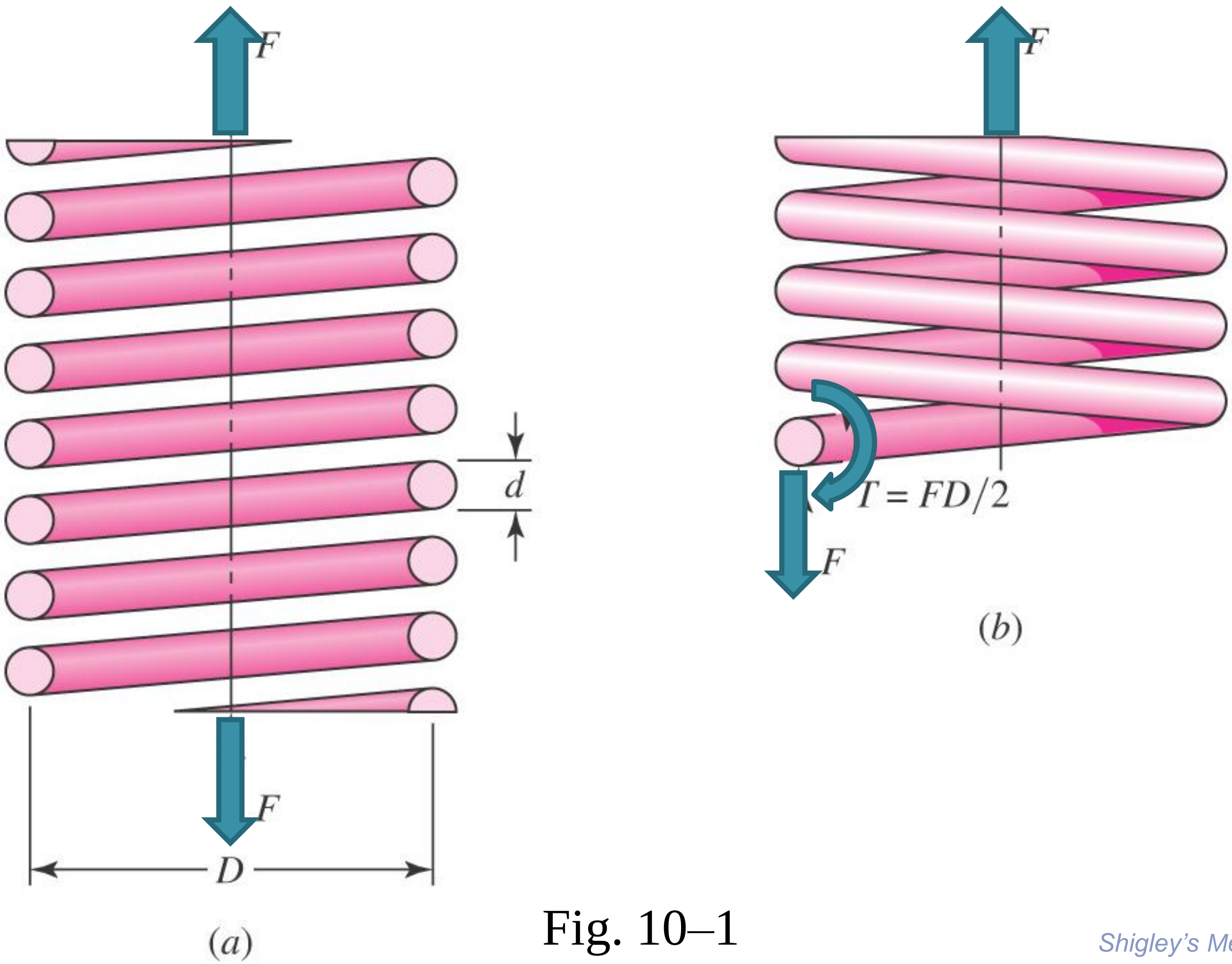


Fig. 10–1

Stresses in Helical Springs

- Torsional shear and direct shear
- Additive (maximum) on inside fiber of cross-section

$$\tau_{max} = \frac{Tr}{J} + \frac{F}{A}$$

- Substitute terms

$$\tau_{max} = \tau, \; T = FD/2, \; r = d/2,$$

$$J = \pi d^4/32 \qquad A = \pi d^2/4$$

$$\tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

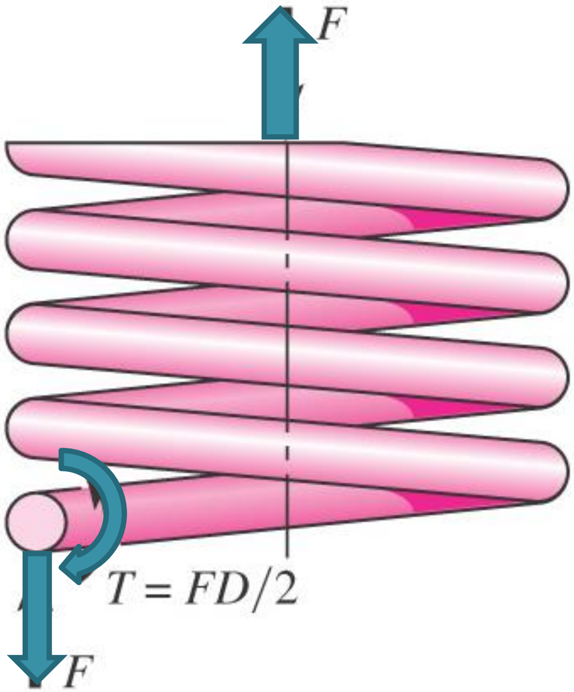


Fig. 10–1*b*

Stresses in Helical Springs

$$\tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Factor out the torsional stress

$$\tau = \left(1 + \frac{d}{2D}\right)\left(\frac{8FD}{\pi d^3}\right)$$

Define *Spring Index* $C = \frac{D}{d}$ (10–1)

Define *Shear Stress Correction Factor*

$$K_s = 1 + \frac{1}{2C} = \frac{2C+1}{2C} \qquad (10\text{–}3)$$

Maximum shear stress for helical spring

$$\tau = K_s \frac{8FD}{\pi d^3} \qquad (10\text{–}2)$$

Curvature Effect

- Stress concentration type of effect on inner fiber due to curvature
- Can be ignored for static, ductile conditions due to localized cold-working
- Can account for effect by replacing K_s with *Wahl factor* or *Bergsträsser factor* which account for both direct shear and curvature effect

$$K_W = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \qquad (10\text{–}4)$$

$$K_B = \frac{4C + 2}{4C - 3} \qquad (10\text{–}5)$$

$$\tau = K_B \frac{8FD}{\pi d^3} \qquad (10\text{–}7)$$

- Cancelling the curvature effect to isolate the curvature factor

$$K_c = \frac{K_B}{K_s} = \frac{2C(4C + 2)}{(4C - 3)(2C + 1)} \qquad (10\text{–}6)$$

Deflection of Helical Springs

$$k \approx \frac{d^4 G}{8 D^3 N}$$

If C >> 1

$$y = \frac{F - F_i}{k}$$

y>0 , only if F>Fi

Initial Tension in Close-Wound Springs

- Initial tension is created by twisting the wire as it is wound onto a mandrel.
- When removed from the mandrel, the initial tension is locked in because the spring cannot get any shorter.
- The amount of initial tension that can routinely be incorporated is shown.
- The two curves bounding the preferred range is given by

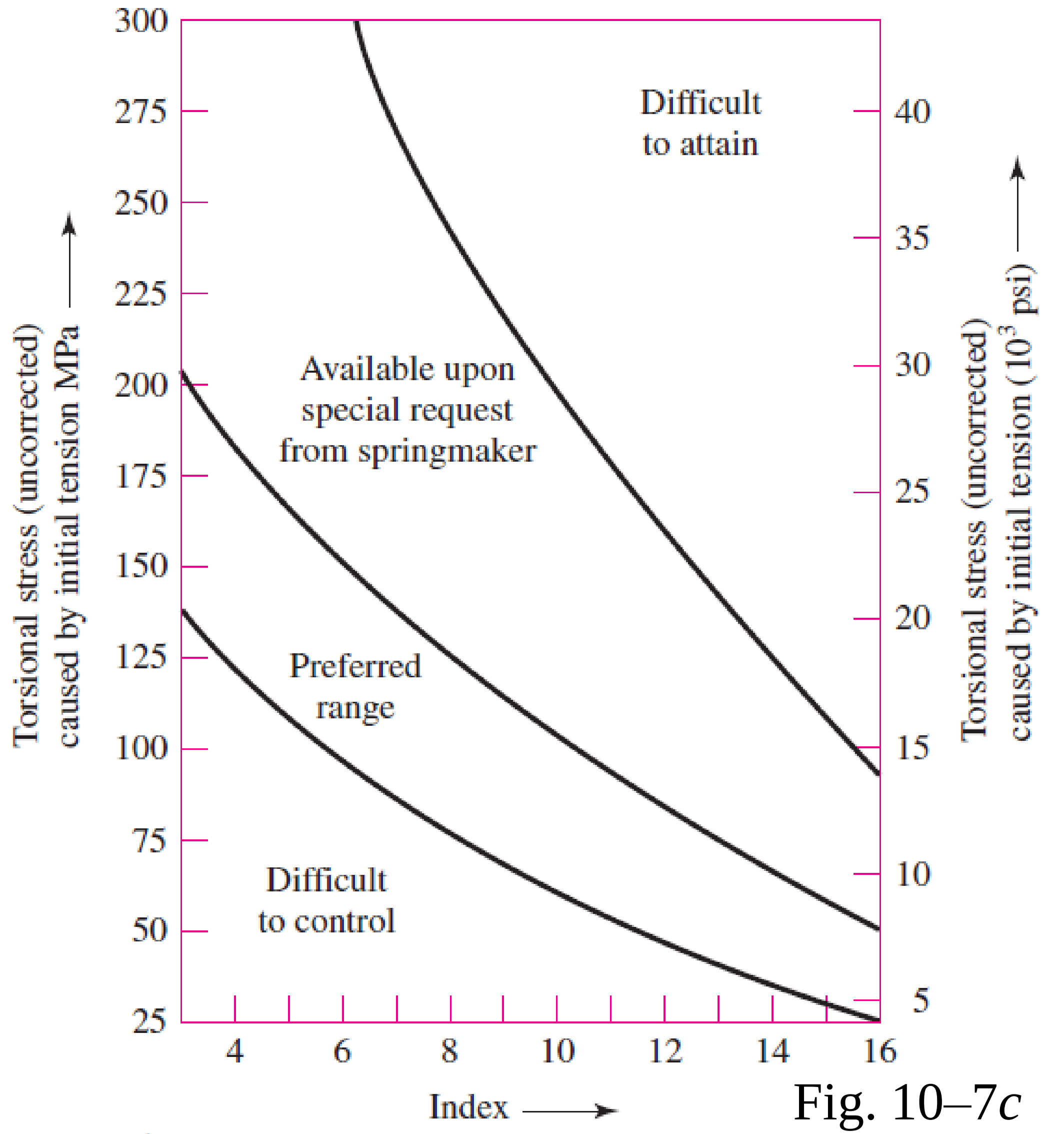


Fig. 10–7*c*

$$\tau_i = \frac{33\ 500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right) \text{psi} \qquad (10\text{–}41)$$

Guidelines for Maximum Allowable Stresses

- Recommended maximum allowable stresses, corrected for curvature effect, for static applications is given in Table 10–7.

Table 10–7

	Percent of Tensile Strength		
	In Torsion		In Bending
Materials	Body	End	End
Patented, cold-drawn or hardened and tempered carbon and low-alloy steels	45–50	40	75
Austenitic stainless steel and nonferrous alloys	35	30	55
	Ssy	Ssy	Sy

This information is based on the following conditions: set not removed and low temperature heat treatment applied. For springs that require high initial tension, use the same percent of tensile strength as for end.

Example 10–6

A hard-drawn steel wire extension spring has a wire diameter of 0.035 in, an outside coil diameter of 0.248 in, hook radii of $r_1 = 0.106$ in and $r_2 = 0.089$ in, and an initial tension of 1.19 lbf. The number of body turns is 12.17. From the given information:

(*a*) Determine the physical parameters of the spring.

(*b*) Check the initial preload stress conditions.

(*c*) Find the factors of safety under a static 5.25-lbf load.

Solution

(*a*)

$$D = \text{OD} - d = 0.248 - 0.035 = 0.213 \text{ in}$$

$$C = \frac{D}{d} = \frac{0.213}{0.035} = 6.086$$

$$K_B = \frac{4C + 2}{4C - 3} = \frac{4(6.086) + 2}{4(6.086) - 3} = 1.234$$

Example 10–6

Eq. (10–40) and Table 10–5:

$$N_a = N_b + G/E = 12.17 + 11.6/28.7 = 12.57 \text{ turns}$$

Eq. (10–9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.035^4(11.6)10^6}{8(0.213^3)12.57} = 17.91 \text{ lbf/in}$$

Eq. (10–39): $$L_0 = (2C - 1 + N_b)d = [2(6.086) - 1 + 12.17]\,0.035 = 0.817 \text{ in}$$

The deflection under the service load is

$$y_{\max} = \frac{F_{\max} - F_i}{k} = \frac{5.25 - 1.19}{17.91} = 0.227 \text{ in}$$

where the spring length becomes $L = L_0 + y = 0.817 + 0.227 = 1.044$ in.

Example 10–6

(*b*) The uncorrected initial stress is given by Eq. (10–2) without the correction factor. That is,

$$(\tau_i)_{\text{uncorr}} = \frac{8F_iD}{\pi d^3} = \frac{8(1.19)0.213(10^{-3})}{\pi(0.035^3)} = 15.1 \text{ kpsi}$$

The preferred range is given by Eq. (10–41) and for this case is

$$(\tau_i)_{\text{pref}} = \frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)$$

$$= \frac{33\,500}{\exp[0.105(6.086)]} \pm 1000\left(4 - \frac{6.086-3}{6.5}\right)$$

$$= 17\,681 \pm 3525 = 21\,206,\ 14\,156 \text{ psi} = 21.2,\ 14.2 \text{ kpsi}$$

Thus, the initial tension of 15.1 kpsi is in the preferred range. Answer

Example 10–6

Thus, the initial tension of 15.1 kpsi is in the preferred range.

(*c*) For hard-drawn wire, Table 10–4 gives $m = 0.190$ and $A = 140 \text{ kpsi} \cdot \text{in}^m$. From Eq. (10–14)

$$S_{ut} = \frac{A}{d^m} = \frac{140}{0.035^{0.190}} = 264.7 \text{ kpsi}$$

For torsional shear in the main body of the spring, from Table 10–7,

$$S_{sy} = 0.45 S_{ut} = 0.45(264.7) = 119.1 \text{ kpsi}$$

The shear stress under the service load is

$$\tau_{max} = \frac{8 K_B F_{max} D}{\pi d^3} = \frac{8(1.234)5.25(0.213)}{\pi(0.035^3)}(10^{-3}) = 82.0 \text{ kpsi}$$

Thus, the factor of safety is

$$n = \frac{S_{sy}}{\tau_{max}} = \frac{119.1}{82.0} = 1.45$$

Answer

Example 10–6

For the end-hook bending at A,

$$C_1 = 2r_1/d = 2(0.106)/0.0.035 = 6.057$$

From Eq. (10–35)

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = \frac{4(6.057^2) - 6.057 - 1}{4(6.057)(6.057 - 1)} = 1.14$$

From Eq. (10–34)

$$\sigma_A = F_{\max}\left[(K)_A \frac{16D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

$$= 5.25\left[1.14\frac{16(0.213)}{\pi(0.035^3)} + \frac{4}{\pi(0.035^2)}\right](10^{-3}) = 156.9 \text{ kpsi}$$

The yield strength, from Table 10–7, is given by

$$S_y = 0.75S_{ut} = 0.75(264.7) = 198.5 \text{ kpsi}$$

The factor of safety for end-hook bending at A is then

$$n_A = \frac{S_y}{\sigma_A} = \frac{198.5}{156.9} = 1.27$$ Answer

Example 10–6

For the end-hook in torsion at B, from Eq. (10–37)

$$C_2 = 2r_2/d = 2(0.089)/0.035 = 5.086$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(5.086) - 1}{4(5.086) - 4} = 1.18$$

and the corresponding stress, given by Eq. (10–36), is

$$\tau_B = (K)_B \frac{8F_{\max} D}{\pi d^3} = 1.18 \frac{8(5.25)0.213}{\pi (0.035^3)}(10^{-3}) = 78.4 \text{ kpsi}$$

Using Table 10–7 for yield strength, the factor of safety for end-hook torsion at B is

$$n_B = \frac{(S_{sy})_B}{\tau_B} = \frac{0.4(264.7)}{78.4} = 1.35$$ Answer

Yield due to bending of the end hook will occur first.

Example 10–7

The helical coil extension spring of Ex. 10–6 is subjected to a dynamic loading from 1.5 to 5 lbf. Estimate the factors of safety using the Gerber failure criterion for (*a*) coil fatigue, (*b*) coil yielding, (*c*) end-hook bending fatigue at point *A* of Fig. 10–6*a*, and (*d*) end-hook torsional fatigue at point *B* of Fig. 10–6*b*.

Solution

A number of quantities are the same as in Ex. 10–6: $d = 0.035$ in, $S_{ut} = 264.7$ kpsi, $D = 0.213$ in, $r_1 = 0.106$ in, $C = 6.086$, $K_B = 1.234$, $(K)_A = 1.14$, $(K)_B = 1.18$, $N_b = 12.17$ turns, $L_0 = 0.817$ in, $k = 17.91$ lbf/in, $F_i = 1.19$ lbf, and $(\tau_i)_{\text{uncorr}} = 15.1$ kpsi. Then

$$F_a = (F_{\max} - F_{\min})/2 = (5 - 1.5)/2 = 1.75 \text{ lbf}$$

$$F_m = (F_{\max} + F_{\min})/2 = (5 + 1.5)/2 = 3.25 \text{ lbf}$$

The strengths from Ex. 10–6 include $S_{ut} = 264.7$ kpsi, $S_y = 198.5$ kpsi, and $S_{sy} = 119.1$ kpsi. The ultimate shear strength is estimated from Eq. (10–30) as

$$S_{su} = 0.67 S_{ut} = 0.67(264.7) = 177.3 \text{ kpsi}$$

Example 10–7

(*a*) Body-coil fatigue:

$$\tau_a = \frac{8K_B F_a D}{\pi d^3} = \frac{8(1.234)1.75(0.213)}{\pi(0.035^3)}(10^{-3}) = 27.3 \text{ kpsi}$$

$$\tau_m = \frac{F_m}{F_a}\tau_a = \frac{3.25}{1.75}27.3 = 50.7 \text{ kpsi}$$

Using the Zimmerli data of Eq. (10–28) gives

$$S_{se} = \frac{S_{sa}}{1 - \left(\dfrac{S_{sm}}{S_{su}}\right)^2} = \frac{35}{1 - \left(\dfrac{55}{177.3}\right)^2} = 38.7 \text{ kpsi}$$

From Table 6–7, p. 315, the Gerber fatigue criterion for shear is

$$(n_f)_{\text{body}} = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(2\frac{\tau_m}{S_{su}}\frac{S_{se}}{\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{177.3}{50.7}\right)^2 \frac{27.3}{38.7}\left[-1 + \sqrt{1 + \left(2\frac{50.7}{177.3}\frac{38.7}{27.3}\right)^2}\right] = 1.24 \quad \text{Answer}$$

Example 10–7

(b) The load-line for the coil body begins at $S_{sm} = \tau_i$ and has a slope $r = \tau_a/(\tau_m - \tau_i)$. It can be shown that the intersection with the yield line is given by $(S_{sa})_y = [r/(r+1)](S_{sy} - \tau_i)$. Consequently, $\tau_i = (F_i/F_a)\tau_a = (1.19/1.75)27.3 = 18.6$ kpsi, $r = 27.3/(50.7 - 18.6) = 0.850$, and

$$(S_{sa})_y = \frac{0.850}{0.850 + 1}(119.1 - 18.6) = 46.2 \text{ kpsi}$$

Thus,

$$(n_y)_{\text{body}} = \frac{(S_{sa})_y}{\tau_a} = \frac{46.2}{27.3} = 1.69$$

Answer

Example 10–7

(*c*) End-hook bending fatigue: using Eqs. (10–34) and (10–35) gives

$$\sigma_a = F_a\left[(K)_A\frac{16D}{\pi d^3}+\frac{4}{\pi d^2}\right]$$

$$= 1.75\left[1.14\frac{16(0.213)}{\pi(0.035^3)}+\frac{4}{\pi(0.035^2)}\right](10^{-3}) = 52.3\text{ kpsi}$$

$$\sigma_m = \frac{F_m}{F_a}\sigma_a = \frac{3.25}{1.75}52.3 = 97.1\text{ kpsi}$$

To estimate the tensile endurance limit using the distortion-energy theory,

$$S_e = S_{se}/0.577 = 38.7/0.577 = 67.1\text{ kpsi}$$

Using the Gerber criterion for tension gives

$$(n_f)_A = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2\frac{\sigma_a}{S_e}\left[-1+\sqrt{1+\left(2\frac{\sigma_m}{S_{ut}}\frac{S_e}{\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{264.7}{97.1}\right)^2\frac{52.3}{67.1}\left[-1+\sqrt{1+\left(2\frac{97.1}{264.7}\frac{67.1}{52.3}\right)^2}\right] = 1.08$$

Answer

Example 10–7

(*d*) End-hook torsional fatigue: from Eq. (10–36)

$$(\tau_a)_B = (K)_B \frac{8F_aD}{\pi d^3} = 1.18\frac{8(1.75)0.213}{\pi(0.035^3)}(10^{-3}) = 26.1 \text{ kpsi}$$

$$(\tau_m)_B = \frac{F_m}{F_a}(\tau_a)_B = \frac{3.25}{1.75}26.1 = 48.5 \text{ kpsi}$$

Then, again using the Gerber criterion, we obtain

$$(n_f)_B = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1+\sqrt{1+\left(2\frac{\tau_m}{S_{su}}\frac{S_{se}}{\tau_a}\right)^2}\,\right]$$

$$= \frac{1}{2}\left(\frac{177.3}{48.5}\right)^2 \frac{26.1}{38.7}\left[-1+\sqrt{1+\left(2\frac{48.5}{177.3}\frac{38.7}{26.1}\right)^2}\,\right] = 1.30$$

Answer

Helical Coil *Torsion* Springs

- Helical coil springs can be loaded with torsional end loads.
- Special ends are used to allow a force to be applied at a distance from the coil axis.
- Usually used over a rod to maintain alignment and provide buckling resistance.

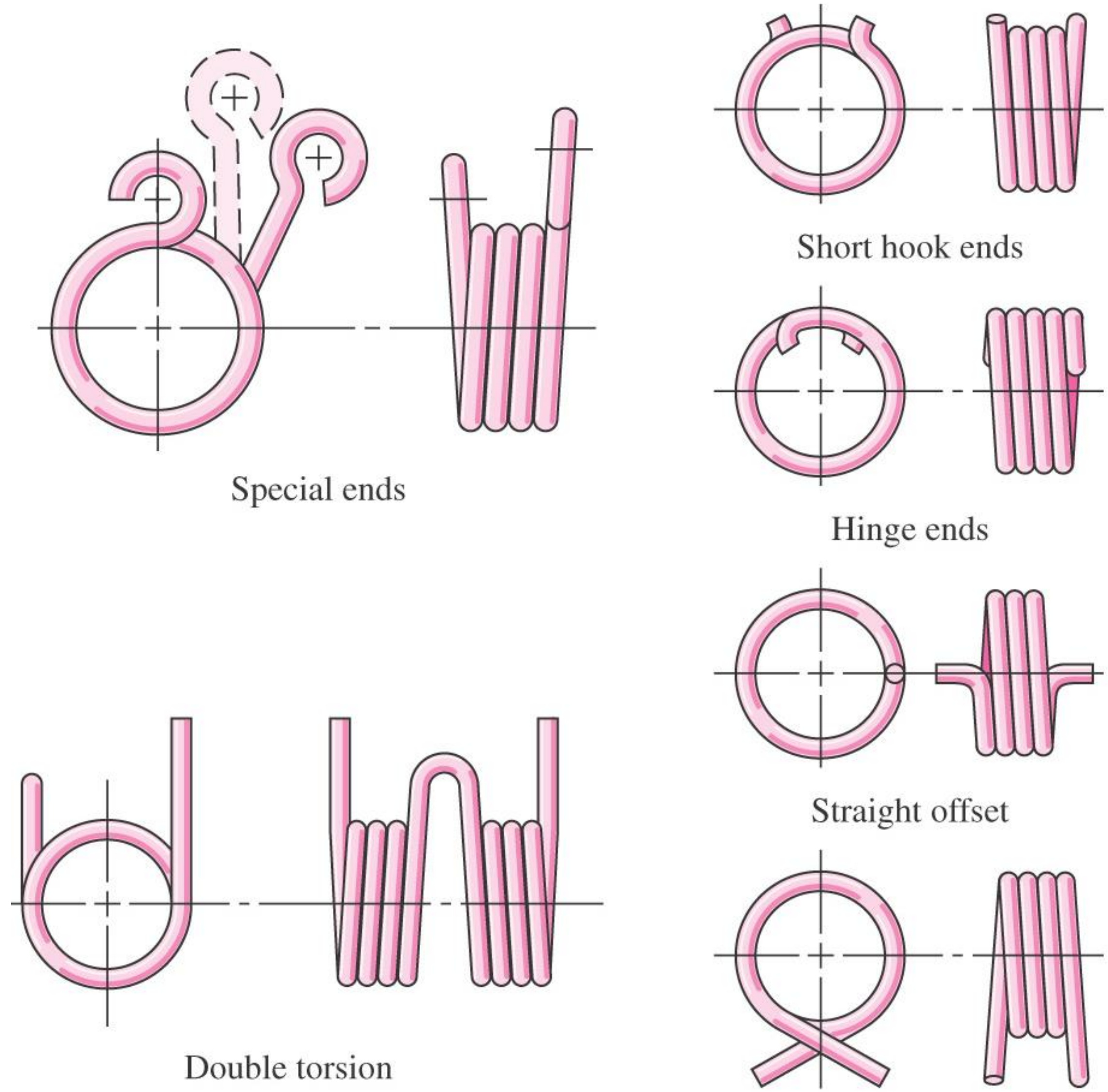


Fig. 10–8

End Locations of Torsion Springs

- Terminology for locating relative positions of ends is shown.
- The initial unloaded partial turn in the coil body is given by

$$N_p = \beta/360°$$

- The number of body turns N_b will be the full turns plus the initial partial turn.

$$N_b = \text{integer} + \frac{\beta}{360°} = \text{integer} + N_p$$

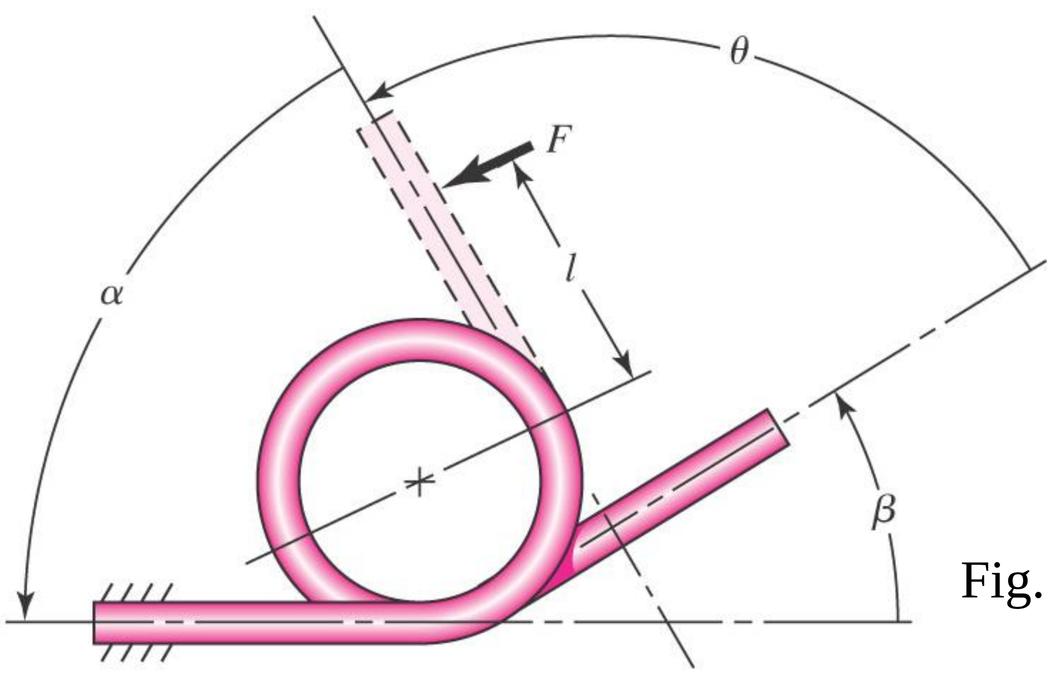


Fig. 10–9

End Locations of Torsion Springs

- Commercial tolerances on relative end positions is given in Table 10–9

Table 10–9

End Position Tolerances for Helical Coil Torsion Springs (for D/d Ratios up to and Including 16)

Source: From *Design Handbook*, 1987, p. 52. Courtesy of Associated Spring.

Total Coils	Tolerance: ± Degrees*
Up to 3	8
Over 3–10	10
Over 10–20	15
Over 20–30	20
Over 30	25

*Closer tolerances available on request.

Stress in Torsion Springs

- The coil of a torsion spring experiences bending stress (despite the name of the spring).
- Including a stress-correction factor, the stress in the coil can be represented by

$$\sigma = K \frac{Mc}{I}$$

- The stress-correction factor at inner and outer fibers has been found analytically for round wire to be

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} \qquad K_o = \frac{4C^2 + C - 1}{4C(C+1)} \qquad (10\text{–}43)$$

- K_i is always larger, giving the highest stress at the inner fiber.
- With a bending moment of $M = Fr$, for round wire the bending stress is

$$\sigma = K_i \frac{32Fr}{\pi d^3} \qquad (10\text{–}44)$$

Spring Rate for Torsion Springs

- Angular deflection is commonly expressed in both radians and revolutions (turns).
- If a term contains revolutions, the variable will be expressed with a prime sign.
- The spring rate, if linear, is

$$k' = \frac{M_1}{\theta'_1} = \frac{M_2}{\theta'_2} = \frac{M_2 - M_1}{\theta'_2 - \theta'_1} \qquad (10\text{–}45)$$

where moment M can be expressed as Fl or Fr.

Deflection in the Body of Torsion Springs

- Use Castigliano's method to find the deflection in radians in the body of a torsion spring.

$$U = \int \frac{M^2\,dx}{2EI}$$

- Let $M = Fl = Fr$, and integrate over the length of the body-coil wire. The force F will deflect through a distance $r\theta$.

$$r\theta = \frac{\partial U}{\partial F} = \int_0^{\pi D N_b} \frac{\partial}{\partial F}\left(\frac{F^2 r^2\,dx}{2EI}\right) = \int_0^{\pi D N_b} \frac{F r^2\,dx}{EI}$$

- Using I for round wire, and solving for θ,

$$\theta = \frac{64 F r D N_b}{d^4 E} = \frac{64 M D N_b}{d^4 E}$$

Deflection in the Ends of Torsion Springs

- The deflection in the ends of the spring must be accounted for.
- The angle subtended by the end deflection is obtained from standard cantilever beam approach.

$$\theta_e = \frac{y}{l} = \frac{Fl^2}{3EI} = \frac{Fl^2}{3E(\pi d^4/64)} = \frac{64Ml}{3\pi d^4 E} \quad (10\text{–}46)$$

Deflection in Torsion Springs

- The total angular deflection is obtained by combining the body deflection and the end deflection.
- With end lengths of l_1 and l_2, combining the two deflections previously obtained gives,

$$\theta_t = \frac{64MDN_b}{d^4E} + \frac{64Ml_1}{3\pi d^4E} + \frac{64Ml_2}{3\pi d^4E} = \frac{64MD}{d^4E}\left(N_b + \frac{l_1+l_2}{3\pi D}\right) \quad (10\text{–}47)$$

Equivalent Active Turns

- The equivalent number of active turns, including the effect of the ends, is

$$N_a = N_b + \frac{l_1 + l_2}{3\pi D} \qquad (10\text{–}48)$$

Spring Rate in Torsion Springs

- The spring rate, in torque per radian

$$k = \frac{Fr}{\theta_t} = \frac{M}{\theta_t} = \frac{d^4 E}{64 D N_a} \quad (10\text{–}49)$$

- The spring rate, in torque per turn

$$k' = \frac{2\pi d^4 E}{64 D N_a} = \frac{d^4 E}{10.2 D N_a} \quad (10\text{–}50)$$

- To compensate for the effect of friction between the coils and an arbor, tests show that the 10.2 should be increased to 10.8.

$$k' = \frac{d^4 E}{10.8 D N_a} \quad (10\text{–}51)$$

- Expressing Eq. (10–47) in revolutions, and applying the same correction for friction, gives the total angular deflection as

$$\theta_t' = \frac{10.8 M D}{d^4 E}\left(N_b + \frac{l_1 + l_2}{3\pi D}\right) \quad (10\text{–}52)$$

Decrease of Inside Diameter

- A torsion spring under load will experience a change in coil diameter.
- If the spring is over a pin, the inside diameter of the coil must not be allowed to decrease to the pin diameter.
- The angular deflection of the body of the coil, extracted from the total deflection in Eq. (10–52), is

$$\theta'_c = \frac{10.8 M D N_b}{d^4 E} \qquad (10\text{–}54)$$

- The new helix diameter D' of a deflected coil is

$$D' = \frac{N_b D}{N_b + \theta'_c} \qquad (10\text{–}53)$$

- The new inside diameter is

$$D'_i = D' - d$$

Decrease of Inside Diameter

- The diametral clearance Δ between the body coil and the pin of diameter D_p is

$$\Delta = D' - d - D_p = \frac{N_b D}{N_b + \theta'_c} - d - D_p \qquad (10\text{–}55)$$

- Solving for N_b,

$$N_b = \frac{\theta'_c(\Delta + d + D_p)}{D - \Delta - d - D_p} \qquad (10\text{–}56)$$

- This gives the number of body turns necessary to assure a specified diametral clearance.

Static Strength for Torsion Springs

- To obtain normal yield strengths for spring wires loaded in bending, divide values given for torsion in Table 10–6 by 0.577 (distortion energy theory). This gives

$$S_y = \begin{cases} 0.78 S_{ut} & \text{Music wire and cold-drawn carbon steels} \\ 0.87 S_{ut} & \text{OQ\&T carbon and low-alloy steels} \\ 0.61 S_{ut} & \text{Austenitic stainless steel and nonferrous alloys} \end{cases} \quad (10\text{–}57)$$

Fatigue Strength for Torsion Springs

- The Sines method and Zimmerli data were only for torsional stress, so are not applicable.
- Lacking better data for endurance limit in bending, use Table 10–10, from Associated Spring for torsion springs with repeated load, to obtain recommended maximum bending stress S_r.

Table 10–10

Maximum Recommended Bending Stresses (K_B Corrected) for Helical Torsion Springs in Cyclic Applications as Percent of S_{ut}

Source: Courtesy of Associated Spring.

Fatigue Life, Cycles	ASTM A228 and Type 302 Stainless Steel		ASTM A230 and A232	
	Not Shot-Peened	Shot-Peened*	Not Shot-Peened	Shot-Peened*
10^5	53	62	55	64
10^6	50	60	53	62

This information is based on the following conditions: no surging, springs are in the "as-stress-relieved" condition.

*Not always possible.

Fatigue Strength for Torsion Springs

- Next, apply the Gerber criterion to obtain the endurance limit.
- Note that repeated loading is assumed.

$$S_e = \frac{S_r/2}{1 - \left(\frac{S_r/2}{S_{ut}}\right)^2} \qquad (10\text{–}58)$$

- This accounts for corrections for size, surface finish, and type of loading, but not for temperature or miscellaneous effects.

Fatigue Factor of Safety for Torsion Springs

- Applying the Gerber criterion as usual from Table 6–7, with the slope of the load line $r = M_a/M_m$,

$$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{r S_{ut}}\right)^2}\right] \qquad (10\text{–}59)$$

$$n_f = \frac{S_a}{\sigma_a} \qquad (10\text{–}60)$$

- Or, finding n_f directly using Table 6–7,

$$n_f = \frac{1}{2}\frac{\sigma_a}{S_e}\left(\frac{S_{ut}}{\sigma_m}\right)^2\left[-1 + \sqrt{1 + \left(2\frac{\sigma_m}{S_{ut}}\frac{S_e}{\sigma_a}\right)^2}\right] \qquad (10\text{–}61)$$

Example 10–8

A stock spring is shown in Fig. 10–10. It is made from 0.072-in-diameter music wire and has $4\frac{1}{4}$ body turns with straight torsion ends. It works over a pin of 0.400 in diameter. The coil outside diameter is $\frac{19}{32}$ in.

(*a*) Find the maximum operating torque and corresponding rotation for static loading.

(*b*) Estimate the inside coil diameter and pin diametral clearance when the spring is subjected to the torque in part (*a*).

(*c*) Estimate the fatigue factor of safety n_f if the applied moment varies between $M_{min} = 1$ to $M_{max} = 5$ lbf · in.

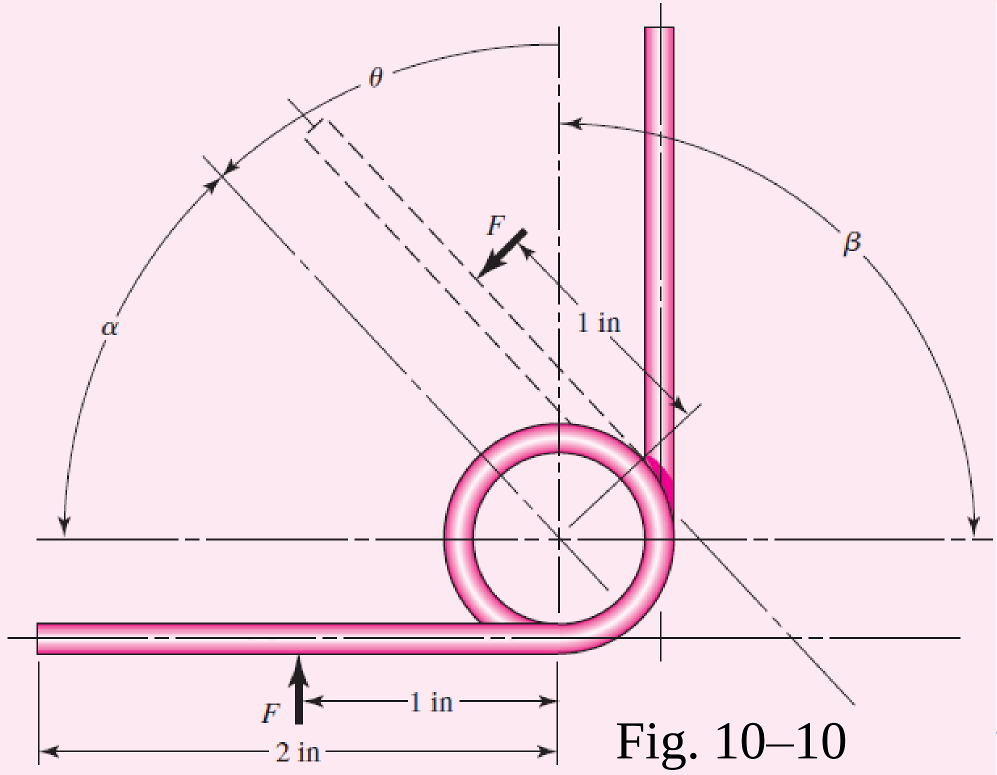


Fig. 10–10

Example 10–8

(*a*) For music wire, from Table 10–4 we find that $A = 201$ kpsi · in^m and $m = 0.145$. Therefore,

$$S_{ut} = \frac{A}{d^m} = \frac{201}{(0.072)^{0.145}} = 294.4 \text{ kpsi}$$

Using Eq. (10–57) gives

$$S_y = 0.78 S_{ut} = 0.78(294.4) = 229.6 \text{ kpsi}$$

The mean coil diameter is $D = 19/32 - 0.072 = 0.5218$ in. The spring index $C = D/d = 0.5218/0.072 = 7.247$. The bending stress-correction factor K_i from Eq. (10–43), is

$$K_i = \frac{4(7.247)^2 - 7.247 - 1}{4(7.247)(7.247 - 1)} = 1.115$$

Now rearrange Eq. (10–44), substitute S_y for σ, and solve for the maximum torque Fr to obtain

$$M_{max} = (Fr)_{max} = \frac{\pi d^3 S_y}{32 K_i} = \frac{\pi (0.072)^3 229\,600}{32(1.115)} = 7.546 \text{ lbf} \cdot \text{in}$$

Example 10–8

Note that no factor of safety has been used. Next, from Eq. (10–54) and Table 10–5, the number of turns of the coil body θ_c' is

$$\theta_c' = \frac{10.8MDN_b}{d^4E} = \frac{10.8(7.546)0.5218(4.25)}{0.072^4(28.5)10^6} = 0.236 \text{ turn}$$

$$(\theta_c')_{\text{deg}} = 0.236(360^\circ) = 85.0^\circ$$ Answer

The active number of turns N_a, from Eq. (10–48), is

$$N_a = N_b + \frac{l_1 + l_2}{3\pi D} = 4.25 + \frac{1+1}{3\pi(0.5218)} = 4.657 \text{ turns}$$

Example 10–8

The spring rate of the complete spring, from Eq. (10–51), is

$$k' = \frac{0.072^4(28.5)10^6}{10.8(0.5218)4.657} = 29.18 \text{ lbf} \cdot \text{in/turn}$$

The number of turns of the complete spring θ' is

$$\theta' = \frac{M}{k'} = \frac{7.546}{29.18} = 0.259 \text{ turn}$$

$$(\theta'_s)_{\text{deg}} = 0.259(360^\circ) = 93.24^\circ$$ Answer

Example 10–8

(*b*) With no load, the mean coil diameter of the spring is 0.5218 in. From Eq. (10–53),

$$D' = \frac{N_b D}{N_b + \theta'_c} = \frac{4.25(0.5218)}{4.25 + 0.236} = 0.494 \text{ in}$$

The diametral clearance between the inside of the spring coil and the pin at load is

$$\Delta = D' - d - D_p = 0.494 - 0.072 - 0.400 = 0.022 \text{ in} \qquad \text{Answer}$$

Example 10–8

(*c*) Fatigue:

$$M_a = (M_{\max} - M_{\min})/2 = (5 - 1)/2 = 2 \text{ lbf} \cdot \text{in}$$

$$M_m = (M_{\max} + M_{\min})/2 = (5 + 1)/2 = 3 \text{ lbf} \cdot \text{in}$$

$$r = \frac{M_a}{M_m} = \frac{2}{3}$$

$$\sigma_a = K_i \frac{32M_a}{\pi d^3} = 1.115 \frac{32(2)}{\pi 0.072^3} = 60\,857 \text{ psi}$$

$$\sigma_m = \frac{M_m}{M_a}\sigma_a = \frac{3}{2}(60\,857) = 91\,286 \text{ psi}$$

Example 10–8

From Table 10–10, $S_r = 0.50S_{ut} = 0.50(294.4) = 147.2$ kpsi. Then

$$S_e = \frac{147.2/2}{1 - \left(\dfrac{147.2/2}{294.4}\right)^2} = 78.51 \text{ kpsi}$$

The amplitude component of the strength S_a, from Eq. (10–59), is

$$S_a = \frac{(2/3)^2 294.4^2}{2(78.51)}\left[-1 + \sqrt{1 + \left(\frac{2}{2/3}\frac{78.51}{294.4}\right)^2}\right] = 68.85 \text{ kpsi}$$

The fatigue factor of safety is

$$n_f = \frac{S_a}{\sigma_a} = \frac{68.85}{60.86} = 1.13$$

Answer